SotFoM III + Hyp

3rd Symposium on the Foundations of Mathematics +
Hyperuniverse Programme Final Conference

21-23 September 2015, Vienna

with the support of the John Templeton Foundation

Organising Committee: Carolin Antos (KGRC), Neil Barton (Birkbeck), Sy-David Friedman (KGRC), Claudio Ternullo (KGRC), John Wigglesworth (MCMP)

Programme

Day 1 (21 September 2015)

10.00-10.05 Introductory Remarks

10.05-11.35 TATIANA AARRIGONI: ‘The Hyperuniverse Program: A Critical Appraisal’

11.35-11.50 Coffee Break

11.50-12.50 GIORGIO VENTURI: ‘Forcing, Multiverse and Realism’

12.50-15.00 Lunch

15.00-16.00 DANIEL WAXMAN and JARED WARREN: ‘Is there a good argument for mathematical pluralism?’

16.00-16.15 Coffee Break

16.15-17.15 MATTEO VIALE: ‘Category forcings and generic absoluteness: Explaining the success of strong forcing axioms’

17.15-17.30 Coffee Break

17.30-19.00 ØYSTEIN LINNEBO: ‘Potentialism about set theory’
Day 2 (22 September 2015)

09.00-09.05 Introductory Remarks
09.00-10.30 MARY LENG: ‘On “Defending The Axioms”’
10.30-10.45 Coffee Break
10.45-12.15 NEIL BARTON: ‘How the hyperuniverse behaves’
12.30 Trip to Mostalm for Social Lunch

15.00-16.00 EMIL WEYDERT: ‘A multiverse axiom induction framework’
16.00-16.15 Coffee Break
16.15-17.15 DOUGLAS BLUE: ‘Forcing axioms and maximality as the demand for interpretability’
17.15-17.30 Coffee Break
17.30-19.00 SY-DAVID FRIEDMAN: ‘What are axioms of set theory?’

Day 3 (23 September 2015)

10.00-10.05 Introductory Remarks
10.05-11.35 GEOFFREY HELLMAN: ‘A height-potentialist multiverse view of set theory’
11.35-11.50 Coffee Break
11.50-12.50 SAM SANDERS: ‘Non-standard analysis as a computational foundation’
12.50-15.00 Lunch
Abstracts

TATIANA ARRIGONI
(Bruno Kessler Foundation)

THE HYPERUNIVERSE PROGRAM: A CRITICAL APPRAISAL

As originally introduced in Friedman-Arrigoni (2012) the Hyperuniverse Program (HP) is a foundational project aimed at extending the realm of set theoretic truths (beyond ZFC) so as to possibly assess independent questions. In its more recent formulation, i.e. Ternullo-Friedman (2014), the HP is said to foster “the search for new set theoretic axioms”, where the latter are explicitly meant as true statements of set theory. In pursuing its overall goal, the HP has developed both philosophical reflections, especially about the nature of set theoretic truth, and mathematical investigations yielding to specific proposals for new truths for set theory. So described, the HP is not alone in the contemporary scenario. Other foundational projects have been put forth in set theory the last few decades sharing, at least prima facie, the same goal as the HP. What is the relation of the HP to these projects? In what the philosophical assumptions lying behind the HP are different from and/or similar to those underlying the latter? Is the background philosophy of HP original or “nothing new under the sun”? In the course of this talk I’ll discuss these and related questions. In doing this, I’ll address some open issues apparently challenging both the novelty and the overall plausibility of the program, and try to show how one may suitably approach them.

GIORGIO VENTURI
(Campinas)

FORCING, MULTIVERSE AND REALISM

In this article we analyze the notion of set theoretical genericity from a more philosophical perspective. After a brief presentation of the method of forcing we outline some of the philosophical imports of this technique in connection with realism. We shall discuss some philosophical reactions to the invention of forcing, concentrating on Mostowski’s proposal of sharpening the notion of generic set. Then we will provide an overview of the notions of multiverse and the related philosophical debate on the foundations of set theory. In conclusion, we connect this modern debate and Mostowski's proposal, suggesting a way to analyze the notion of genericity within the framework of a multiverse structure.

JARED WARREN AND DANIEL WAXMAN
(NYU)

IS THERE A GOOD ARGUMENT FOR MATHEMATICAL PLURALISM?

Many philosophers of mathematics are attracted to pluralist views of set theory, according to which there are several, equally acceptable set theories that are prima facie incompatible with one another. There is, furthermore, a widespread belief that pluralism is motivated in some way or another by the current state of set theory — most especially, the vast range of independence results made possible by the success of modern set theoretical techniques such as forcing. Despite this situation, however, it is surprisingly difficult to find a coherent and persuasive argument from these results to pluralism. In this talk, we discuss three different varieties of arguments for pluralism: epistemological (according to which pluralism
is required in order to avoid debilitating epistemological problems; naturalist (according to which pluralism is justified by attention to the practice of set theory) and metaconceptual (according to which pluralism is the result of an indeterminacy of our concept of set). We reconstruct a version of the metaconceptual argument, and argue that it is the most plausible motivation for pluralism.

MATTEO VIALE
(TURIN)

CATEGORY FORCINGS AND GENERIC ABSOLUTENESS: EXPLAINING THE SUCCESS OF STRONG FORCING AXIOMS

We present two arguments which outline why forcing axioms have been so successful in resolving most of the undecidability issues at the core of the current investigations in set theory. The first argument follows from a deep observation of Todorcevic and outlines that forcing axioms can be introduced as maximal strengthenings of the axiom of choice: likewise this axiom they are extremely useful to construct mathematical objects which cannot be exhibited by means of purely constructive methods. The second argument is based on my recent work on generic absoluteness and its relation with forcing axioms and outlines that these axioms provide a very useful complete semantic for third order number theory: in short in order to prove that a third order arithemetic statement follows from a given forcing axiom it is enough to produce (by means of appropriate forcings) a model in which the statement holds. In particular forcing axioms transform validity problems (which are quite hard being universal statements) into consistency problems (which are generally easier being existential statements).

OYSTEIN LINNEBO
(OSLO)

POTENTIALISM ABOUT SET THEORY

According to potentialism, the set theoretic hierarchy cannot be regarded as a completed whole. For however many sets have been formed, it is possible to form more. This view is explored and compared with the ancient Aristotelian conception of potential infinity. Special attention is paid to the question of which logical and mathematical principles distinguish potentialism from actualism.

MARY LENG
(YORK)

ON ‘DEFENDING THE AXIOMS’

Penelope Maddy’s 2011 book, *Defending the Axioms*, argues that there is a fact of the matter about whether there are sets whose cardinality is strictly less than the real numbers, but strictly greater than the natural numbers, but that there is no fact of the matter about whether there are sets. I will question Maddy’s first claim (which rests on Maddy’s interpretation of mathematical practice), but agree with Maddy on the metaphysical claim that, once the position she calls ‘Robust Realism’ (which she herself defended in earlier work) is ruled out, the issue between realism and anti-realism in the philosophy of mathematics dissolves into a merely terminological dispute.
**NEIL BARTON**  
*(BIRKBECK)*

**HOW THE HYPERUNIVERSE BEHAVES**

Friedman’s Hyperuniverse project has represented a controversial yet foundationally fruitful approach to the search for new set-theoretic axioms. There are many and varied facets of the view, but two key tenets are (1.) The synthesising of various maximality criteria, and (2.) The analysis of these criteria in the collection of all countable transitive models. In this paper, I analyse how the tools of the Hyperuniverse can be put to work on different views concerning Potentialism and Actualism in set-theoretic ontology and reference. We shall see that different combinations of Potentialism and Actualism mesh with varying levels of success with respect to (1.) and (2.). The results are surprising; for many features of the Hyperuniverse, Actualism with respect to both height and width represents a more natural setting for justifying axioms in the spirit of (1.). However, we shall also see that the status of (2.) is problematic on such a picture, and the necessity of this part of the programme will be challenged.

**GEOFFREY HELLMAN**  
*(MINNESOTA)*

**A HEIGHT-POTENTIALIST MULTIVERSE VIEW OF SET THEORY**

We first review the basics of the modal-structural approach to mathematics, concentrating on second-order ZFC set theory. (We draw inspiration here from Zermelo’s great 1930 paper). The use of modal operators is critical in successfully expressing the extendability principle that Zermelo sought to articulate. Moreover, it leads to an improved resolution (we submit) of the set-theoretic paradoxes, illustrated briefly with the Burali-Forti. We then describe how the framework generates the small large cardinals through the indescribables, via second-order reflection, but without appeal to “absolute infinity”. Finally, time permitting, we propose a simplification of the original modal translation (due to Putnam), as applied to unbounded second-order set-theoretic sentences.

**SAM SANDERS**  
*(MCMP)*

**NONSTANDARD ANALYSIS AS A COMPUTATIONAL FOUNDATION**

The program Univalent foundations of mathematics is usually described by its proponents as follows. Vladimir Voevodsky’s new program for a comprehensive, computational foundation for mathematics based on the homotopical interpretation of type theory. The subliminal message here is clear: The ‘old’ foundation of mathematics, namely Zermelo Fraenkel set theory with the axiom of choice (ZFC), has the distinct disadvantage of not being computational in nature, whence the new foundation seems ‘better’ than the old one. Nonetheless, we establish in this paper that Nonstandard Analysis enriches ZFC with a similar (but hitherto unknown) computational foundation. In other words, Univalent Foundations does not necessarily have a ‘computational edge’ over ZFC, thanks to Nonstandard Analysis.
A MULTIVERSE AXIOM INDUCTION FRAMEWORK

The multiverse paradigm in set theory does not only reflect philosophical preferences, or set up a new playground for mathematical investigation, but it also offers a powerful methodological tool for investigating the conceptual foundations of set theory by guiding the search for and the evaluation of new set-theoretic axioms or facts. The prototypical example is Friedman's Hyperuniverse Program (HUP). Our goal is to develop an abstract inferential framework generalizing the HUP whose (defeasible, inductive) inference methods are meant to identify or validate new axioms. We consider nonmonotonic consequence relations $\vdash$, parametrized by specifications of the multiverse and set-theoretic desiderata, which associate with any suitable $\text{ZFC} + X$ new - not necessarily classically derivable - candidate truths. The desiderata could, for instance, consist of consistency conditions or maximization demands w.r.t. preorders over universes. There are a number of possible inductive strategies, but it doesn't seem that conceptual considerations at the level of set theory are sufficient to decide among them. The idea is therefore to also assess the inferential level and to use rationality postulates for nonmonotonic inference, heavily investigated within AI for modeling commonsense reasoning, to classify and evaluate such procedures. This is however a non-trivial task because of the special characteristics of axiom induction.

FORCING AXIOMS AND MAXIMALITY AS THE DEMAND FOR INTERPRETABILITY

Combinatorial maximality is often invoked to justify forcing axioms. This form of maximality is a notoriously imprecise notion in the context of set theory. I claim that maximality should be understood as the demand that a candidate axiom maximize interpretative power. In this light, maximality does not provide justification for forcing axioms. I proceed to give an alternative possible justification of Martin’s Maximum. I argue that $\text{MM}$ allows one to effectively complete power sets in a manner analogous to how $(\ast)$ completes $\mathcal{P}(\omega_1)$. This naturally gives rise to the conjecture that MM and $(\ast)$ are compatible. I conclude, however, that even this conjecture, if true, does not provide substantial evidence for MM. Forcing axioms are still in need of a philosophical basis.

WHAT ARE AXIOMS OF SET THEORY?

What qualifies a statement of set theory to gain reference as an “axiom”? Set-theorists use this term loosely and for several different purposes. In the talk I'll discuss the different uses of the term “axiom” in set theory and try to dispel some of the ongoing confusion about the meaning of set-theoretic truth.
Location

The KGRC is located on the top (second) floor of the north wing of the historical Josephinum Building in Währingerstraße 25, Vienna. On the map the building is marked by a red pin.

Trams of lines 37–42 run past the KGRC approximately every 3 minutes during core hours. The unidirectional Sensengasse tram stop (blue on the map) is only a few metres from the KGRC. All southbound trams towards Schottentor in the inner city centre stop here. (Schottentor is also a U2 metro stop, marked by a blue train in the south of the map.) Trams in the other direction do not stop at Sensengasse. Get off at the following stop (also blue on the map) and walk back the short distance.

Enter the Josephinum court/garden through the right-hand side gate. Turn right and enter the right-hand side wing, which houses a medical history museum on the first floor. Take the stairs past the museum to the second/top floor. (The KGRC is also accessible via a lift through the Josefinum's central main entrance, but there is a slight chance that you might encounter a locked door at the end of that route.)

Travel Information

Vienna International Airport is located in Schwechat, just outside the city border. In the local transport association VOR, it is in zone 280, whereas the city center forms the core zone 100. The cheapest connection to the city center is the regular S7 train, which runs every 30 minutes roughly from 5 am to midnight. The price of a single ticket for the two zones is 4.40 Euros. There are ticket machines near the platform. (It is possible, but may be tricky, to buy a 24-hour ticket or other season ticket for Vienna from the ticket machines in Schwechat. If you do so and you validate it right away, you can save 2.20 Euros by combining it with a 1-zone ticket just for zone 280. Conversely, if you already have any ticket valid for the core zone 100, then to get to the airport you only need a single ticket for zone 280, which it is possible though tricky to buy from the ticket machines in Vienna.)

The City Airport Train (CAT) also connects the airport to the city center. It runs on the same tracks as the S7 trains and has the same frequency. The CAT stops in the city center around the same time as the S7, but is 9 minutes faster. If you take the CAT, you will need another ticket for the remainder of your journey in Vienna. Normally, it is only worth taking the CAT if you are in a rush to get to the airport or want to make use of the unusual and convenient City Check-In service to dispose of your luggage before spending a few more hours in the city center. This service is available from some, but not all, airlines.

Though the S7 trains and the CAT share the same tracks, neither accepts the other's tickets and they use different platforms. Signage for the CAT is more obvious, especially at the airport. If you want to take the S7, initially follow the signs to the CAT, but watch out for the stylized white S (three straight strokes) on a blue background.
The bus service Vienna Airport Lines is only worth considering if you are staying close to Schwedenplatz/Morzinplatz or must be at the airport in the middle of the night.

The taxi fare is about 25–30 Euros and should be negotiated in advance with the driver because the journey crosses the border between the states of Vienna and Lower Austria. Bratislava Airport (Slovakia) is not far from Vienna and used by some cheap operators. Munich Airport (Germany) and Budapest Airport (Hungary) are also in a somewhat reasonable distance from Vienna.

**Public Transport**

If you plan to use public transportation often the weekly ticket is the best option. It is valid from Monday 0.00 until the next Monday 9.00 in the morning.

You can buy your tickets at a ticket machine in any metro station or S-Bahn station or from a tobacconist. You cannot buy them at tram stops or bus stops. Only single tickets are available from the small ticket machines inside trams. At the other ticket machines you can generally choose whether you want your ticket pre-validated for immediate use. If you buy an unvalidated ticket, make sure not to forget stamping it before use, as ticket inspections are very frequent and the fine is 100 Euros. At most ticket machines you can select any of the following languages: German, English, French, Russian and a few others.
Kurt Gödel Research Center

Information for visitors to the Kurt Gödel Research Center. Visitors typically stay at the Hotel Boltzmann.
via Währinger Str. and Boltzmanngasse 5 min
Show terrain 350 m
Details